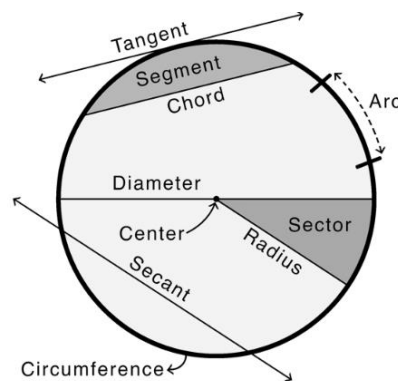# CHAPTER -12

AREAS RELATED TO CIRCLE

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MAIN CONCEPTS

A closed, round geometric figure in which the set of all the points in the plane is equidistant from a given point called 'centre'.

Parts of a circle:-

Perimeter of a Circle:- The distance covered by travelling once around a circle is its perimeter, usually called its circumference. It is **$2\pi r$**.
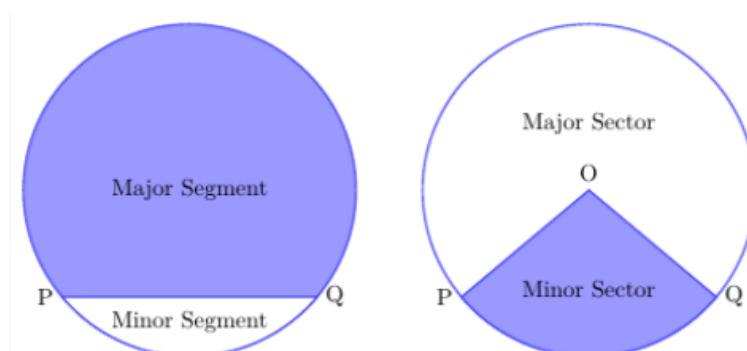
Area of a Circle:- The area enclosed by a circle of radius r is **πr^2**.

- **π is equal to $\frac{22}{7}$ or 3.14 depending on the given question.**

Area of the circular path formed by two concentric circles of radii r_1 and r_2 ($r_1 > r_2$)

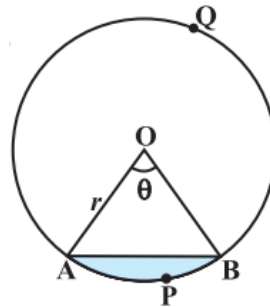$$= \pi r_1^2 - \pi r_2^2 = \pi(r_1^2 - r_2^2)$$

Sector and Segment of a Circle:-

- ➢ **Area of the minor sector of angle θ** $= \dfrac{\theta}{360} \times \pi r^2$
- ➢ **Area of major sector of angle θ** $= \pi r^2 -$ Area of the minor sector
- ➢ **Length of an arc of a sector of angle θ** $= \dfrac{\theta}{360} \times 2\pi r$

where r is the radius of the circle and θ the angle of the sector in degrees.

- • **θ is called the central angle.**

Areas of Segment of a Circle:-

- ➢ **Area of the segment APB** = Area of the sector OAPB – Area of Δ OAB

$$= \dfrac{\theta}{360} \times \pi r^2 - \text{area of OAB}$$

- ➢ **Area of major segment AQB** $= \pi r^2 -$ Area of the minor segment APB

SOLVED QUESTIONS

***1. If the perimeter of the circle and square are equal, then the ratio of their areas will be equal to:

    (a) 14:11   (b) 22:7   (c) 7:22   (c) 11:14

    Ans:- **(a) 14:11**

    Explanation: Given,

    The perimeter of circle = perimeter of the square

    $2\pi r = 4a$

    $a = \pi r/2$

    Area of square $= a^2 = (\pi r/2)^2$

    $A_{circle}/A_{square} = \pi r^2/(\pi r/2)^2$

    $= 14/11$

*2. The area of a sector of a circle with radius 6 cm if the angle of the sector is 60° will be.

(a) 142/7cm²   (b) 152/7 cm²   (c) 132/7 cm²   (d) 122/7 cm²

Ans:- **(c) 132/7 cm²**

       Explanation: Angle of the sector is 60°

       Area of sector = $(\theta/360°) \times \pi r^2$

       ∴ Area of the sector with angle 60° = $(60°/360°) \times \pi r^2$ cm²

       = $(36/6) \pi$ cm²

       = $6 \times (22/7)$ cm²

       = 132/7 cm²

*3. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. The length of the arc is;

(a) 20cm   (b) 21cm   (c) 22cm   (d) 25cm

Ans"- **(c) 22cm**

     Explanation: Length of an arc = $(\theta/360°) \times (2\pi r)$

     ∴ Length of an arc AB = $(60°/360°) \times 2 \times 22/7 \times 21$

     = $(1/6) \times 2 \times (22/7) \times 21$

     Or, Arc AB Length = 22cm

*4. If the radius of a circle is 4.2 cm, compute its area and circumference.

       Ans:- Area of a circle = πr^2

     So, area = $\pi(4.2)^2$ = 55.44 cm²

     Circumference of a circle = $2\pi r$

     So, circumference = **$2\pi(4.2)$ = 26.4 cm**

**5. Find the area of the sector of a circle with a radius of 4cm and of angle 30°. Also, find the area of the corresponding major sector.

     Ans :- Radius = r = 4 cm, θ=30°
     Area of sector = $[\theta/360] \times \pi r^2$
     = $30/360 \times 3.14 \times (4)^2$
     = $1/12 \times 3.14 \times 4 \times 4$
     = $1/3 \times 3.14 \times 4$
     = 12.56/3 cm²
     = 4.19 cm²
     Area of major sector = $((360 - \theta)/360) \times \pi r^2$

$= ((360 − 30))/360×3.14×(4)^2$

$= 330/360×3.14×4×4$

$= 11/12×3.14×4×4$

$= 46.05 \text{ cm}^2$

***6. A wheel has diameter 84 cm. Find how many complete revolutions must it take to cover 792 meters.

Ans:- Let r be the radius of the wheel. Then,

Diameter = 84 cm

2r=84

r=42 cm

Circumference of the wheel = $2\pi r=264cm=2.64m$

So, the wheel covers 2.64 meters in one complete revolution.

Total number of revolutions in covering 792 meters = 792/2.64=300.

Hence, the wheel takes 300 revolutions in covering 792 meters.

**7. A chord AB of a circle of radius 10 cm makes a right angle at the centre of the circle. Find the area of the minor and major segments.

Ans:- Area of segment APBA = area of sector OAPB−area of ΔOAB

=$θ/(360°)×\pi r^2−1/2×OB×OA$

=$(90°)/(360°)×22/7×10×10−1/2×10×10$

=$550/7-50 = (550-530)/7 = 200/7$

=$28.5cm^2$
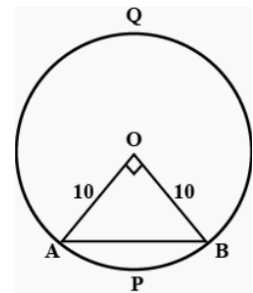
Area of major segment = $\pi r^2−28.5=22/7×10×10−28.5$

= $2200/7−28.5=(2200−199.5)/7=200.5/7$

= 285.5 cm^2

***8. If a square is inscribed in a circle, find the ratio of the areas of the circle and the square.

Ans:- Let ABCD be a square inscribed in a circle of radius 'r'. Now, the diameter of circle is the diagonal of square.
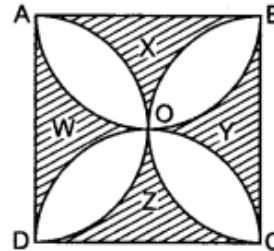
Therefore, BD=2r. In △BDC, using Pythagoras theorem

$BC^2+CD^2=BD^2⇒a^2+a^2=(2r)^2⇒2a^2=4r^2⇒a^2=2r^2$

∴Area of square=2r²

Area of circle=πr²

Required ration = πr² : 2r² = π: : 2

***9. In figure, ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region.**

 Ans:- Area of the square ABCD = 14 x 14 = 196 cm²

Area of semicircle AOB=1/2 x πr²

=1/2×22/7x7x7

Similarly, area of semicircle DOC = 77 cm²

Hence, the area of shaded region (Part W and Part Y) = Area of square -Area of two semicircles AOB and COD

= 196 – 154 = 42 cm²

Therefore, area of four shaded parts (i.e. X, Y, W, Z) = (2 x 42) cm² = 84 cm²

10. A chord subtends an angle of 90°at the centre of a circle whose radius is 20 cm. Compute the area of the corresponding major segment of the circle.

 Ans :- Point to note:

Area of the sector = θ/360 × π × r²

Base and height of the triangle formed will be = radius of the circle

Area of the minor segment = area of the sector – area of the triangle formed

Area of the major segment = area of the circle – area of the minor segment

Now,

Radius of circle = r = 20 cm and

Angle subtended = θ = 90°

Area of the sector = θ/360 × π × r² = 90/360 × 22/7 × 20²

Or, area of the sector = 314.2 cm²

Area of the triangle = ½ × base × height = ½ × 20 × 20 = 200 cm²

Area of the minor segment = 314.2 – 200 = 114.2 cm²

Area of the circle = π × r² = (22/7) × 20² = 1257.14

Area of the major segment = 1257.14 – 114.2 = 1142 .94 cm²

So, the area of the corresponding major segment of the circle = 1142 .94 cm²

***11. Calculate the perimeter of an equilateral triangle if it inscribes a circle whose area is 154 cm²

Ans: - Here, as the equilateral triangle is inscribed in a circle, the circle is an incircle.

Now, the radius of the incircle is given by,

r = Area of triangle/semi-perimeter

In the question, it is given that area of the incircle = 154 cm²

So, π × r² = 154

Or, r = 7 cm

Now, assume the length of each arm of the equilateral triangle to be "x" cm

So, the semi-perimeter of the equilateral triangle = (3x/2) cm

And, the area of the equilateral triangle = (√3/4) × x²

We know, r = Area of triangle/semi-perimeter

So, r = [x²(√3/4)/ (3x/2)]

=> 7 = √3x/6

Or, x = 42/√3

Multiply both numerator and denominator by √3

So, x = 42√3/3 = 14√3 cm

Now, the perimeter of an equilateral triangle will be = 3x = 3 × 14√3 = 72.7 cm.

***12. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Ans :- The radius of car's wheel = 80/2 = 40 cm (as D = 80 cm)

So, the circumference of wheels = $2\pi r = 80\,\pi$ cm

Now, in one revolution, the distance covered = circumference of the wheel = $80\,\pi$ cm

It is given that the distance covered by the car in 1 hr = 66km

Converting km into cm we get,

Distance covered by the car in 1hr = (66×10^5) cm

In 10 minutes, the distance covered will be = $(66 \times 10^5 \times 10)/60 = 1100000$ cm/s

∴ Distance covered by car = 11×10^5 cm

Now, the no. of revolutions of the wheels = (Distance covered by the car/Circumference of the wheels) = $11 \times 105\ /80\ \pi = 4375$.

***13. Mr Ramanand purchased a plot QRUT to build his house. Following is scaled down figure of his plot. He leave space of two congruent semicircles for gardening and a rectangular area of breadth 3 cm for car parking.
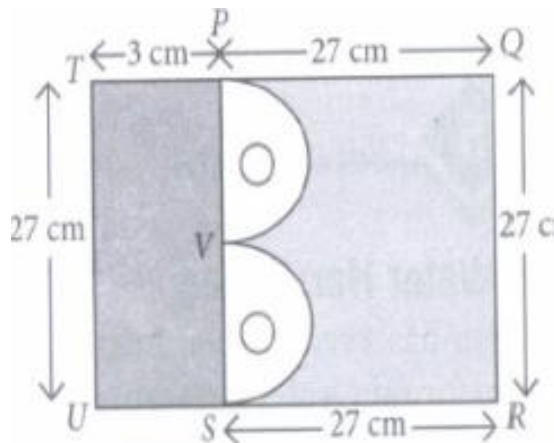
Based on the above information, answer the following questions. Use $\pi = 3.14$

(i) Area of square PQRS is

    (a) 700 cm^2 (b) 729 cm^2 (c) 732 cm^2 (d) 735 cm^2

Ans:- (b) 729 cm^2. Area of square = 27 x 27 = 729cm^2

(ii) Area of rectangle left for car parking is

    (a) 64 cm^2 (b) 76 cm^2 (c) 81 cm^2 (d) 100 cm^2

Ans:- (c) 81 cm^2. Area of Rectangle = 3x27= 81cm^2

(iii) Radius of semi-circle is

    (a) 6.75 cm   (b) 7 cm   (c) 7.75 cm   (d) 8.75 cm

Ans:- (a) 6.75 cm. Diameter of semi-circle = 27/2 = 13.5cm. Radius of circle = 13.5/2 = 6.75cm

(iv) Find the area of the shaded region

    (a) 660.82 cm^2 (b) 666.72 cm^2 (c) 669.89 cm^2 (d) 700 cm^2

Ans:- (b) 666.72 cm^2. Area of shaded part= area of plot – area of two semi- circles.

= 27x30 – 3.14 x (6.75)^2 = 666.72cm^2

PRACTICE QUESTIONS

SECTION A

MULTIPLE CHOICE QUESTIONS:

**1. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

    (a) 2 units   (b) π units   (c) 4 units   (d) 7 units

*2. Area of a sector of angle p (in degrees) of a circle with radius R is

    (a) p/180 × $2\pi R$ (b) p/180 × πR^2

    (c) p/360 × $2\pi R$ (d) p/720 × $2\pi R^2$

***3. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then

    (a) $R_1 + R_2 = R$ (b) $R_1^2 + R_2^2 = R^2$

    (c) $R_1 + R_2 < R$ (d) $R_1^2 + R_2^2 < R^2$

***4. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be

    (a) 10 m   (b) 15 m   (c) 20 m   (d) 24 m

*5. The area of a quadrant of a circle with circumference of 22 cm is

    (a) 77 cm^2 (b) 77/8 cm^2 (b) 35.5 cm^2 (c) 77/2 cm^2

**6. In a circle of radius 14 cm, an arc subtends an angle of 30° at the centre, the length of the arc is

(a) 44 cm   (b) 28 cm   (c) 11 cm   (d) 22/3 cm

***7. The length of the minute hand of a clock is 14 cm. The area swept by the minute hand in 5 minutes   is

(a) 153.9 cm²   (b) 102.6 cm²   (c) 51.3 cm²   (d) 205.2 cm²

***8. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is

(a) 56 cm   (b) 42 cm   (c) 28 cm   (d) 16 cm

***9. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm respectively, is

(a) 31 cm   (b) 25 cm   (c) 62 cm   (d) 50 cm

***10. The wheel of a motor cycle is of radius 35 cm. How many revolutions per minute must the wheel make so as to keep a speed of 66 km/h?

(a) 300   (b) 400   (c) 450   (d) 500

***11. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20m × 16m, then the area of the field in which the cow can graze is:

(a) 154 m^2 (b) 156 m^2 (c) 158 m^2 (d) 160 m^2

**12. A pendulum swings through on angle of 30∘ and describes an arc 8.8 cm in length. Find the length of pendulum in cm.

(a) 16.8   (b) 17.3   (c) 15.1   (d) 14.5

*13. Radius of the outer circle is 18 cm and the radius of the inner circle is 7 cm. What is the area of the region between the outer and the inner circles?

(a) 361 π cm^2 (b) 133 cm^2 (v) 192.5 cm^2 (d) 275 π cm^2

**14. A wire is bent to form a circle of radius 7 cm. From the resulting shape, a chunk of the wire is cut off, and the wire cut off measures 4 cm in length. The length of the remaining wire is

(a) 45cm   (b) 50cm   (c) 40cm   (d) 42cm

*15. In the figure, the area of the outer ring iS

(a) Area of outer circle + Area of inner circle

(b) Area of outer circle – Area of inner circle

(c) Area of inner circle – Area of outer circle

(d) Area of outer circle
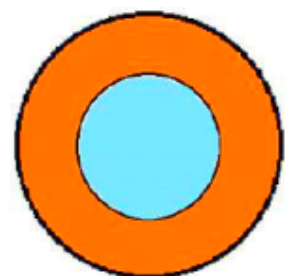
SECTION B

2 MARKS QUESTIONS:-

**1. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

***2. If the area of a circle is equal to sum of the areas of two circles of diameters 10 cm and 24 cm, calculate the diameter of the larger circle (in cm).

***3. If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = 22/7$, calculate the circumference (in cm) of the circle.
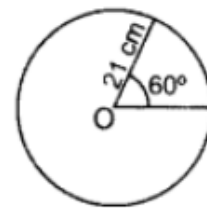
**4. Observe the following figure
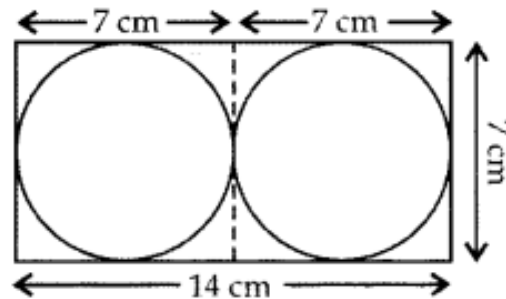
Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular card board of dimensions 14 cm × 7 cm. Find the area of the remaining card board. [Use $\pi = 22/7$]
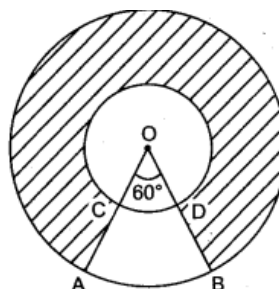
**5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc

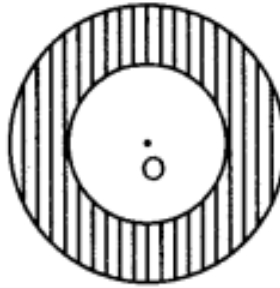(ii) area of the sector formed by the arc. [Use $\pi = 22/7$]

***6. In figure, two concentric circles with centre O, have radii 21 cm and 42 cm. If ∠AOB = 60°, find the area of the shaded region
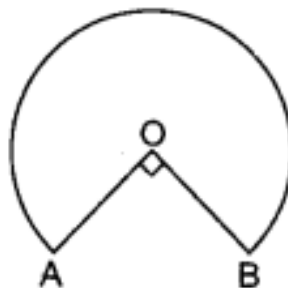
**7. In the given figure, the area of the shaded region between two concentric circles is 286 cm^2.If the difference of the radii of the two circles is 7 cm, find the sum of their radii.


**8. In the given figure, the shape of the top of a table is that a sector of a circle with centre O and ∠AOB = 90°. If AO = OB = 42 cm, then find the perimeter of the top of the table

*9. In figure, PQ and AB are respectively the arcs of two concentric circles of radii 7 cm and 3.5 cm and centre O. If ∠POQ = 30°, then find the area of the shaded region.
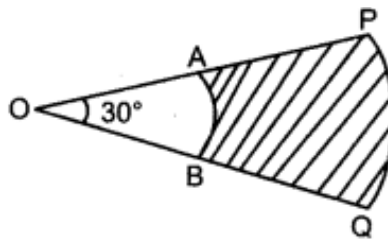
***10. In given figure, a semicircle is drawn with O as centre and AB as diameter. Semicircles are drawn with AO and OB as diameters.If AB = 28 m, find the perimeter of the shaded region.


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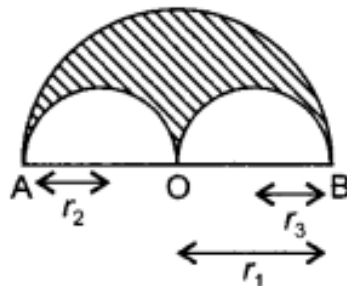
SECTION C

THREE MARKS QUESTIONS:

**1. A chord of length 10 cm divides a circle of radius $5\sqrt{2}$ cm in two segments. Find the area of the minor segment.

**2. Find the area of the major segment APB in a circle of radius 35 cm and $\angle AOB = 90°$.

***3. The circumference of a circle exceeds its diameter by 180 cm. Then find its radius.

***4. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60cm, calculate the speed in km/h.

***5. The circumference of a circular park is 314m. a 20m wide track runs around it. Calculate the cost of laying turf in the park at Rs. 1.25 per sq.m. and the cost of the concrete track at Rs1.25sp.m.

**6. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20m × 16m. Find the area of the field in which the cow cannot graze.

**7. A circular park is surrounded by a road 21m wide. If the radius of the park is 105m, find the area of the road.

**8. A circular pond is of diameter 17.5m is surrounded by a 2 m wide path. Find the cost of constructing the path at a rate of Rs 25 per m^2.

*9. Find the area of the sector of a radius 5cm, if the corresponding arc length is 3.5cm.

**10. In figure, ABCD is a trapezium with AB|| DC, AB=18cm, DC=32cm and the distance between AB and DC is t14cm. if arcs of eqal radii 7cm have been drawn, with centres A,B,C, and D, then find the shaded portion.
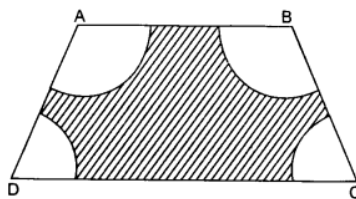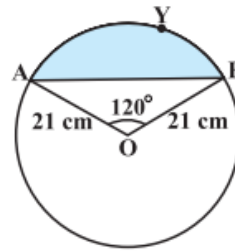

SECTION D

FIVE MARKS QUESTIONS:

***1. A round table cover has six equal designs like segments between two radii. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2. (Use$\sqrt{3} = 1.7$)

***2. Find the area of the segment AYB shown in Figure, if radius of the circle is 21 cm and ∠ AOB = 120°. (Use π = 22/ 7)

**3. In Fig. there are three semicircles, A, B and C having diameter 3 cm each, and another semicircle E having a circle D with diameter 4.5 cm are shown. Calculate the cost of painting the shaded region at the rate of ₹25 per cm^2

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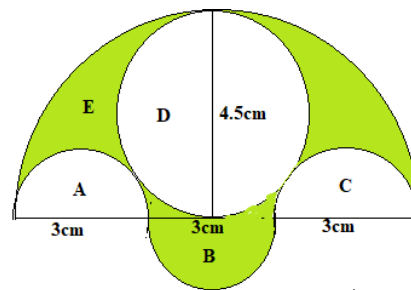
***4. With the vertices A, B and C of a triangle ABC as centres, arcs are drawn with radii 5 cm each as shown in Fig. 11.15. If AB = 14 cm, BC = 48 cm and CA = 50 cm, then find the area of the shaded region. (Use π = 3.14).
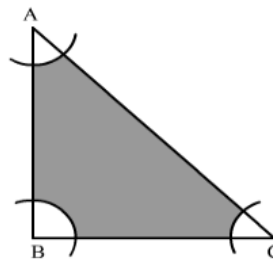
**5. A calf is tied with a rope of length 6 m at the corner of a square grassy lawn of side 20 m. If the length of the rope is increased by 5.5m, find the increase in area of the grassy lawn in which the calf can graze.

**6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC


in the middle as shown in figure. Find the area of the design.

***7. Sides of a triangular field are 15 m, 16 m and 17 m. With the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7m each to graze in the field. Find the area of the field which cannot be grazed by the three animals.

***8. The diameters of front and rear wheels of a tractor are 80 cm and 2m respectively. Find the number of revolutions that rear wheel will make in covering a distance in which the front wheel makes 1400 revolutions.

***9. An archery target has three regions formed by three concentric circles as shown in figure. If the diameters of the concentric circles are in the ratio 1:2:3, then find the ratio of the areas of three regions.
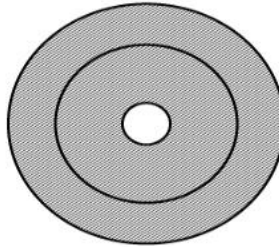
*10. Find the difference of the areas of the two segments of a circle formed by a chord of length 5cm subtending an angle of 90° at the centre.

CASE BASED QUESTIONS:

***1. There is a race competition between all students of a sports academy, so that the sports committee can chose better students for a marathon. The race track in the academy is in the form of a ring whose inner most circumference is 264 m and the outer most circumference is 308m.
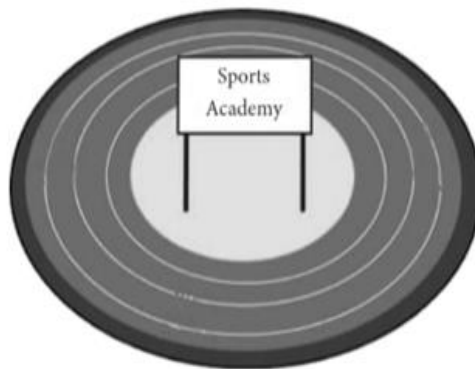

Based on the above information, answer the following questions:-

(i) Find the radius of the outermost circle.

(ii) Find the area of the race track.

(iii) If the cost of painting on the race track is Rs. 6 per m^2, then find the total cost for the painting of the whole race track.

**2. Sarita held a Japanese fan in her hand as shown in the figure below. It is shaped like a sector of a circle and made a thin material such as paper or feather. The inner and outer radii are 3cm and 5cm respectively. The fan has three colours i.e., red, blue, green.

Based on the above information, answer the following questions:-
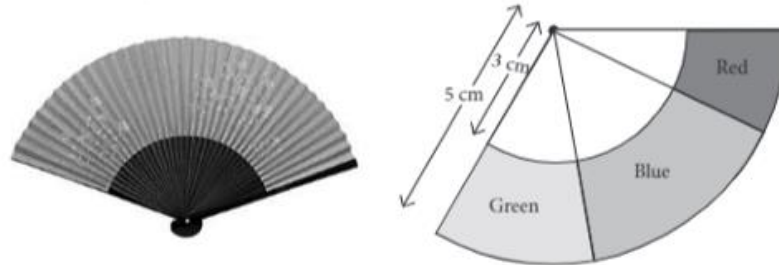
(i) If the region containing blue colour makes an angle of 80° at the centre, then find the area of the region having blue colour.

(ii) If the region containing green colour makes an angle of 60° at the centre, then find the area of the region having green colour.

(iii) Find the area of the whole region having radius 3cm if the central angle made by red region is 20°.

*3. While dusting a maid found a button whose upper face is of black colour as shown in the figure. The diameter of each of the smaller identical circles is ¼ of thee diameter of the larger circle whose radius is 16cm.

Based on the above information, answer the following questions:-

(i) Find the area of each of the smaller circle?

(ii) Find the area of black colour region?

(iii) Find the area of quadrant of the smaller circle?

*****4. Director of a company selects a round glass trophy for awarding their employees on annual function. Design of each trophy is made as shown in the figure, where it's base ABCD is golden plated from the front side at the rate of Rs 6 per cm². **
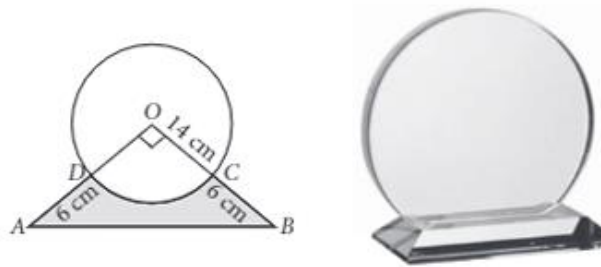
Based on the above information, answer the following questions:-

   (i) Find the area of the sector ODCO.

   (ii) Find the total cost of gold plating.

   (iii) Find the length of arc DC.

**5. Shweta wants to change the design of the floor of her living room which is of the dimension 6m X 4m and it is covered with circular tiles of diameter 50cm each as shown below.
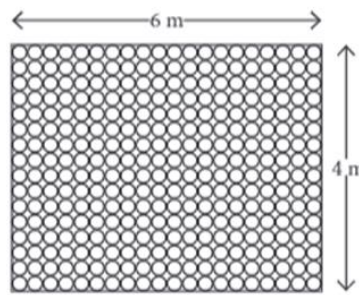
Based on the above information, answer the following questions:-

   (i) Find the number of circular tiles along length of room.

   (ii) Find the total number of tiles in the floor.

   (iii) Find the area of the floor that remains uncovered by the tiles.

ANSWWR KEY CHAPTER 12 AREA RELATED TO CIRCLE

SECTION A MCQ ANSWERS:-

1. (a) 2 units   2. (d) p/720 × 2πR^2 3. **(b) R$_1^2$ + R$_2^2$ = R^2**   4. **(a) 10 m**

5. **(b) 77/8 cm^2**   6. **(d) 22/3 cm**   7. (c) 51.3 cm²   8. (c) 28 cm

9 (d) 50 cm   10. (d) 500   11. (a) 154 m^2 12. (a) 16.8   13. (d) 275 π cm^2

14. (c) 40cm   15. (b) Area of outer circle – Area of inner circle

SECTION B 2 MKS ANSWERS:-

1. 10cm   2. 26 cm   3. 44cm   4. 21cm^2 5. (i) 22cm   (ii) 231cm^2

6. 3465cm^2 7. 13cm   8. 282cm   9. 9.625 cm^2 10. 88m

SECTION C 3 MKS ANSWERS:-

1. 14.25cm^2 2. 3500cm^2 3. 42cm   4. 15.84km/h   5. Rs.9812.5, Rs9420

6. 166 m^2 7. 15246 m^2 8. Rs 3061.50   9. 8.75cm^2 10. 196 m^2

SECTION D ANSWERS:-

1. Rs. 162.66   2. $\frac{21}{4}(88 - 21\sqrt{3})$ 3. Rs. 309   4. 296.75 cm^2 5. 75.625 m^2

6. $(\frac{22528}{7} - 768\sqrt{3})$ cm^2 7. $(24\sqrt{21} - 77)$m^2 8. 560   9. 1:3:5

10. 32.1429 cm^2

CASE STUDY BASED ANSWERS:-

1. (i) 49m   (ii) 2002m^2 (iii) Rs. 12012

2. (i) 11.17cm^2 (ii) 8.38cm^2 (iii) 12.57 cm^2

3. (i) 50.28cm^2 (ii) 603.45cm^2 (iii) 12.57cm^2

4. (i) 154cm^2 (ii) Rs. 276   (iii) 22cm

5. (i) 50   (ii) 8   (iii) 5.142 m^2

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